## Railway Traffic Management

The Meet \& Pass Problem
Pedro A. Afonso \& Carlos F. Bispo
Instituto de Sistemas e Robótica Instituto Superior Técnico
Lisbon, Portugal

## Outline

- Introduction
- Overview
- The approach
- Results
- Conclusions


## Introduction

- Railway companies aim to achieve regular and reliable train services.
- Daily schedules are produced offline to meet such objectives.
- During execution, events may disturb the original schedule.
- Such disturbances are more dramatic in the context of single track lines for outbound and inbound trains.


## Introduction

- For such lines, the original schedule accounts for Meet and Pass points at sidings or stations.
- Two trains meet while traveling in opposite directions;
- A faster train needs to pass a slower train ahead.
- A disturbance during the schedule execution may compromise one or many such points.
- Whenever that happens, an alternative has to be produced in real time.



## Introduction

- A conflict is said to occur whenever two trains are bound to share the same track segment.
- By definition of a track segment, at most one train may travel it;
- Meet conflict - If two trains approach each other on a single track segment, traveling in opposite directions;
- Pass conflict - If a faster train catches a slower train traveling in the same direction on the same track segment.


## Introduction

- Typpically, conflicts are solved by human operators.
- Decisions have to be produced in a timely manner, instructing each of the conflicting trains with what to do.
- A given conflict resolution may induce future conflicts.
- Train priorities is usually the criterion to decide on each conflict.



## Introduction

- The quality of these decisions may not be the best.
- Operators rely on experience;
- Operators have no decision support system;
- Operators may only forecast the impact of their decisions on a relatively small time frame with very simple graphic applications.
- Therefore, there is an opportunity to develop a Decision Support System.



## Overview



## The approach

- The focus of this presentation will be on the "Conflict Resolution" box.
- Formulating a mathematical model;
- Charaterizing the conflict detection problem;
- Identifying conflicts and their solutions;
- Implementation

Beijing Jiaotong University, China

## The approach - Mathematical formulation

- Objective function

$$
\min Z=\sum_{i=1}^{n} w_{i} \max \left\{0,\left(a_{i}^{m}-\alpha_{i}^{m}\right)\right\}
$$

- Subject to:
- Free running time constraints

$$
r_{i}^{k} \geq \tau_{i}^{k}, \forall i \in I, k=1,2, \ldots, m-1
$$

- Consecutive departure and arrival constraints

$$
f_{i}^{k} \geq s_{i}^{k}+\tau_{i}^{k}, \forall i \in I, k=1,2, \ldots, m-1
$$

## The approach - Mathematical formulation

- Subject to (continued):
- Minumum dwell time constraints

$$
s_{i}^{u} \geq \omega_{i}^{u}, \forall i \in I, u=1,2, \ldots, m
$$

- Headway constraints on arrival times to stations

$$
a_{i}^{u} \geq a_{i^{\prime}}^{u}+g_{u} \oplus a_{i^{\prime}}^{u} \geq a_{i}^{u}+g_{u}, \forall i, i^{\prime} \in I, i \neq i^{\prime}, u \in U
$$

- Meet condition

$$
d_{i}^{u+1} \geq a_{i}^{u+1}+g_{u} \oplus d_{i}^{u} \geq a_{i}^{u}+g_{u}, \forall u \in U, i \in I_{i}, i^{\prime} \in I_{o}
$$

## The approach - Mathematical formulation

- Subject to (continued):
- Pass condition

$$
\left(d_{i}^{u} \leq d_{i^{\prime}}^{u}+h^{k} \wedge a_{i}^{u+1} \leq a_{i^{\prime}}^{u+1}+h^{k}\right)
$$

$\oplus$

$$
\left(d_{i^{\prime}}^{u} \leq d_{i}^{u}+h^{k} \wedge a_{i^{\prime}}^{u+1} \leq a_{i}^{u+1}+h^{k}\right)
$$

$$
\forall u \in U,\left\{i, i^{\prime}\right\} \in I_{0}
$$

- Meetpoint capacity limits*

$$
S^{u} \leq c^{u}, \forall u \in U
$$

## The approach - Conflict detection

- Theorem 1
- If train $i$ does not collide with train $i+1$ and train $i+1$ does not collide with train $i+2$, then train $i$ cannot collide with train $i+2$.
- Theorem 2
- If there is a conflict between train $i$ and train $p$, with $p \geq i+2$, there there is also a conflict between trains $i$ and $p-1$ or between trains $p-1$ and $p$.


## The approach - Conflict detection

- Corollary
- In order to conclude about the existence, or nonexistence of conflicts, in a given track segment, it is only necessary to check for conflicts between consecutive trains, in terms of their entering order.
- Concluding:
- The conflict detection is linear, instead of quadratic, in the number of trains.



## The approach - Conflict resolution

- Meet Conflict



## The approach - Conflict resolution

## - Pass Conflict



## The approach - Conflict resolution

- Safety intervals at stations Conflict



## The approach - Conflict resolution

- Capacity Conflict - example for 3 trains with a 2 train capacity


Beijing Jiaotong University, China

## The approach - Implementation



## The approach - Implementation

## Heuristic Solution

- Applies train priorities, as a human controller would do, until it produces a conflict free schedule.
- When trains have the same priority, uses the FOFI dispatching rule.
- First Out First Serve;
- Also known as First Leave First Serve.


## Flowchart



## The approach - Implementation

## Search-based solution

- Performs a Depth First Search.
- Produces complete conflict free schedules fast.
- Improves over them, using best known schedule to bound the search.
- Branch-and-bound.
- Has a computation time budget, after which it returns the best solution found.
- May produce optimal schedules.


## Flowchart



## The approach - Implementation

- The first time a conflict is detected, priority is given to produce one first conflict free schedule. ${ }^{-}$This is not much different from what human operators do.
- After that, knowing how much time is has until the first decision has to be enforced, uses that time to improve over the first solution.
- When time expires, either it produces a better solution or the first is executed.



## Results

- The final package possesses a series of parameters, which affect its performance.
- Initial schedule
- offline schedule for the day;
- Time horizon
- no conflicts up to the horizon;
- Number of solutions
- solutions presented to the human operator as alternatives;
- Cost function
- metric used to evaluate the effects of delays for trains;
- Maximum search time
- computational budget;
- Upper bound
- heuristic schedule



## Results

## Initial Schedule

－Evaluating performance as a function of the initial schedu－ le＇s complexity and number of conflicts．

| 亚 | 关 |  |  | CPU Time（s） |  | Weighted Tardiness |  | Limits |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | 首 | 磁 | 㖪 | 离淢 |
| 1 | 6 | 3 | 8 | 2，63 | 2，58 | 37，35 | 23，80 | 79 | 429 |
| 2 | 12 | 6 | 11 | 2，52 | 3，97 | 101，55 | 44，30 | 10 | 353 |
| 3 | 12 | 24 | 22 | 3，01 | 944，89 | 46，54 | 43，72 | 62 | 141694 |
| 4 | 20 | 24 | 38 | 2，73 | 1800＊ | 52，91 | 52，14 | 330 | 250591 |
| 5 | 40 | 24 | 240 | 6，09 | 1800＊ | 321,9 | 382,27 | 669 | 205350 |

## Number of solutions

－Number of solutions provided to the human dispatcher as alternative solutions for the same conflict．

| Input <br> Schedules | Number <br> of <br> Solutions | CPU Time（s） <br> Optimal <br> Solution |
| :---: | :---: | :---: |
| 3 | 1 | 27,88 |
|  | 5 | 30,78 |
|  | 10 | 32,79 |
| 4 | 1 | 29,67 |
|  | 5 | 33,39 |
|  | 10 | 35,16 |

## Results

## Time horizon

- How far in time does the search provide a conflict free schedule.
- How does that affect performance.

| Input <br> Schedules | Time Horizon <br> (h) | CPU Time (s) |  | Weighted Tardiness |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Optimal <br> Solution | Heuristic <br> Solution | Optimal <br> Solution |  |
| 3 | 2 | 2,62 | 2,44 | 1,754 | 1,754 |
|  | 5 | 2,81 | 2,74 | 9,822 | 9,822 |
|  | 10 | 3,72 | 27,88 | 25,30 | 24,61 |
|  | 24 | 3,01 | 944,89 | 46,54 | 43,72 |
|  | 2 | 2,90 | 2,42 | 1,762 | 1,762 |
|  | 5 | 2,63 | 2,73 | 11,734 | 11,734 |
|  | 10 | 3,09 | 29,67 | 29,53 | 26,72 |
|  | 24 | 2,73 | $1800^{*}$ | 52,91 | 52,14 |
|  | 2 | 3,17 | 2,9 | 0,438 | 0,438 |
|  | 2 | 2,93 | $1800^{*}$ | 70,42 | 64,31 |
|  | 5 | 3,53 | $1800^{*}$ | 152,64 | 186,80 |
|  | 10 | 6,09 | $1800^{*}$ | 321,90 | 382,27 |
|  | 24 |  |  |  |  |

## Results

## Cost function

## Comparison

- Sets of weights for the weighted tardiness function

|  | Priority 1 | Priority 2 | Priority 3 |
| :--- | :---: | :---: | :---: |
| Set 1 | 0.7 | 0.2 | 0.1 |
| Set 2 | 0.6 | 0.3 | 0.1 |
| Set 3 | 0.5 | 0.4 | 0.1 |
| Set 4 | 0.5 | 0.3 | 0.2 |


| Input <br> Schedules | Cost Function | Sets of Weights | Solution Cost |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | Optimal | Heuristic |
| 4 | Weighted Tardiness | Set 1 | 23,97 | 24,66 |
|  |  | Set 2 | 29,97 | 31,27 |
|  |  | Set 3 | 31,76 | 37,88 |
|  |  | Set 4 | 27,23 | 30,00 |
|  | Total tardiness | - | 69,47 | 94,52 |
| 5 | Weighted Tardiness | Set 1 | 182,07 | 169,92 |
|  |  | Set 2 | 170,94 | 174,50 |
|  |  | Set 3 | 158,93 | 179,14 |
|  |  | Set 4 | 161,52 | 209,23 |
|  | Total tardiness | - | 433,75 | 771,02 |

## Results

## Maximum search time

- Performance achieved with progressively larger computational budgets.

| Input <br> Schedules | Maximum Search Time (s) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1 5}$ | $\mathbf{3 0}$ | 60 | $\mathbf{3 0 0}$ | 600 | 1800 |  |
|  | 49,73 | 49,22 | 48,12 | 46,29 | 46,29 | $43,72^{*}$ |  |
|  | 59,79 | 56,22 | 55,72 | 54,12 | 54,12 | 52,14 |  |
|  | 386,22 | 386,12 | 385,84 | 385,62 | 382,27 | 382,27 |  |

## Maximum search time

- Ratio* to optimal solution



## Results

## Initial upper bound

- Effect of using the heuristic solution as starting upper bound for the search.

| Input <br> Schedules | Initial <br> Upper <br> Bound | $\mathbf{1 5}$ | $\mathbf{3 0}$ | $\mathbf{6 0}$ | $\mathbf{3 0 0}$ | $\mathbf{6 0 0}$ | $\mathbf{1 8 0 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Infinite | 49,73 | 49,22 | 48,12 | 46,29 | 46,29 | 43,72 <br> $(944 \mathrm{~s})$ |
|  | Heuristic <br> Solution $=$ <br> 46,54 | 46,54 | 46,54 | 46,54 | 46,54 | 46,54 | 43,72 <br> $(903 \mathrm{~s})$ |
| 4 | Infinite | 59,79 | 56,22 | 55,72 | 54,12 | 54,12 | 52,14 |
|  | Heuristic <br> Solution= <br> 52,91 | 52,91 | 52,91 | 52,91 | 52,91 | 52,91 | 51,95 |

Beijing Jiaotong University, China

## Conclusions

- Presented a Decision Support System for Railway Traffic Management.
- Combines what human operators do with a complementary search engine;
- Provides more than a solution to be chosen;
- Takes advantage of the time to the next conflict to improve over a first heuristic solution;
- Always produces a solution fast.


## Conclusions

- Caracterized the computational complexity for conflict detection.
- Conflicts addressed
- The meet conflict;
- The pass conflict;
- Safety intervals conflict;
- Capacity conflict;
- Presented a series of numerical results to evaluate main features.



## Conclusions

- Future work
- Need to address networks of lines, instead of a single line;
- Move from single track to multiple tracks;
- There are no meet points in multiple tracks.
- But there may be connecting trains that need to be synchronized at given stations



# Thank You 

谢谢您



